A Power Balanced Time-stepping Finite Element Method for Transient Magnetic Field Computation

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Abstract — Conventional transient finite element methods (FEM) of magnetic field and circuit coupled problems may result in unbalanced power computation. The coreloss computation is usually in a post processing so that its effect to other physical quantities cannot be included. This paper mainly studies the power balanced computation in FEM from two important aspects, which are, namely the time integration algorithm and inclusion of the coreloss computation in system equations. A novel approach is proposed, which can remain the balanced power in time integration process and include the coreloss effect in the FEM system equations. Consequently, the input power, losses and output power of electric devices is balanced and the accuracy of the solutions is guaranteed. An example of a laminated transformer is calculated to verify the effectiveness of the proposed method.

I. INTRODUCTION

In order to reduce induced eddy-current losses, the iron cores in electromagnetic devices are usually designed to be laminated. A precise estimation of coreloss is important and necessary for the laminated transformer and electric machine device designs [1-3]. The existing approach for coreloss computation is usually computed in a post-processing and its effect is not included in the system equations of FEM in time domain [4-5]. Consequently, the input power, plus the losses which have not taken the coreloss into account, and the output power of the system, which has to include the coreloss, can not be balanced and hence the efficiency of system is not able to be computed accurately. In addition this paper points out that some most widely used time integration methods, for example, backward Euler method, is not a power balanced algorithm itself. Crank-Nicolson method is power balanced. However it may lead to unstable if the time step size is too large.

In this paper, a power balanced FEM formulation is deduced to take into account the coreloss effect of laminated iron cores into the system equations of the transient FEM. A power balanced and stable time integration is carried on to avoid the numerical error of unbalanced power from the time integration algorithm itself. The advantage is that the input power increase caused by coreloss effect can be precisely computed; no power unbalance in the time integration process is caused and hence the power in the system remains balanced.

II. FORMULATIONS

A. Power Balanced Time Integration Method

For simplicity, a simple a.c. voltage driven resistance R and inductance L series-connected electric circuit is used to explain our analysis. The voltage balance equation of the circuit is:

$$v = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t}\cdot \tag{1}$$

Multiplying current i on the two sides of the equation, we have the power balance equation:

$$vi = Ri^2 + Li\frac{\mathrm{d}i}{\mathrm{d}t}\cdot \tag{2}$$

The average power in one period T is:

$$\frac{1}{T}\int_0^T vi\mathrm{d}t = \frac{1}{T}\int_0^T Ri^2\mathrm{d}t + \frac{1}{T}\int_0^T Li\frac{\mathrm{d}i}{\mathrm{d}t}\mathrm{d}t\cdot \tag{3}$$

For steady-state solution with periodic excitation, the average reactive power in one period T is:

$$\frac{1}{T}\int_0^T Li\frac{\mathrm{d}i}{\mathrm{d}t}\mathrm{d}t = \frac{1}{T}\frac{L}{2}\left[i^2(T) - i^2(0)\right] = 0\cdot \tag{4}$$

If using backward Euler method, the equation (1) is discretized as:

$$v^k = Ri^k + L\frac{i^k - i^{k-1}}{\Delta t}\cdot \tag{5}$$

Multiplying i^k to the two sides of the above equation, we have

$$v^k i^k = R(i^k)^2 + Li^k\frac{i^k - i^{k-1}}{\Delta t}\cdot \tag{6}$$

The average reactive power in one period is:

$$\frac{L}{T}\int_0^T i^k\frac{i^k - i^{k-1}}{\Delta t}\mathrm{d}t = \frac{L}{T\Delta t}\int_0^T \left(i^k i^k - i^k i^{k-1}\right)\mathrm{d}t \neq 0\cdot \tag{7}$$

It means, the algorithm itself is not power balanced.

However, if we use Crank-Nicolson method, we have

$$L\frac{i^k - i^{k-1}}{\Delta t} = \frac{v^{k-1} - Ri^{k-1}}{2} + \frac{v^k - Ri^k}{2}\cdot \tag{8}$$

That is

$$L\frac{i^k - i^{k-1}}{\Delta t} = \frac{v^{k-1} + v^k}{2} - \frac{R\left(i^{k-1} + i^k\right)}{2}. \tag{9}$$

Multiplying $\frac{i^k + i^{k-1}}{2}$ to the two sides of the above equation, we have:

$$L\frac{(i^k)^2 - (i^{k-1})^2}{2\Delta t} = \left(\frac{v^{k-1} + v^k}{2}\right)\left(\frac{i^{k-1} + i^k}{2}\right) - R\left(\frac{i^{k-1} + i^k}{2}\right)^2\cdot \tag{10}$$

The average reactive power in one period T is:

$$\frac{L}{T}\int_0^T \frac{(i^k)^2 - (i^{k-1})^2}{2\Delta t}\mathrm{d}t = \frac{L}{2T\Delta t}\left[\int_{\Delta t}^{T+\Delta t}\left(i^{k-1}\right)^2\mathrm{d}t - \int_0^T\left(i^{k-1}\right)^2\mathrm{d}t\right] = 0\cdot \tag{11}$$

It means that the algorithm of Crank-Nicolson method itself is power balanced.

Backward Euler method is unconditionally stable.

However Crank-Nicolson method may lead to unstable if the time step size is too large. To overcome this disadvantage, we properly control the time step size to prevent the unstable solution. The FEM can lead to solve the algebraic matrix equation [6]:

$$\boldsymbol{C}\boldsymbol{X} + \boldsymbol{D}\frac{\partial \boldsymbol{X}}{\partial t} = \boldsymbol{P}\,. \tag{12}$$

Usually the coefficient matrix $\boldsymbol{C}$ is dependent on time and the solution $\boldsymbol{X}$. The coefficient matrix $\boldsymbol{D}$ is constant. Using Crank-Nicolson method we have its recurrence formula:

$$\left(\boldsymbol{C}^k + \frac{2\boldsymbol{D}}{\Delta t}\right)\boldsymbol{X}^k = \boldsymbol{P}^k + \boldsymbol{P}^{k-1} + \left(-\boldsymbol{C}^{k-1} + \frac{2\boldsymbol{D}}{\Delta t}\right)\boldsymbol{X}^{k-1}. \tag{13}$$

If we use backward Euler method but use $\Delta t/2$ as the step size, we have its recurrence formula:

$$\left[\boldsymbol{C}^k + \frac{2\boldsymbol{D}}{\Delta t}\right]\boldsymbol{X}_*^k = \boldsymbol{P}^k + \frac{2\boldsymbol{D}}{\Delta t}\boldsymbol{X}^{k-1/2}, \tag{14}$$

where

$$\boldsymbol{X}^{k-1/2} = \frac{\boldsymbol{X}^{k-1} + \boldsymbol{X}^k}{2}. \tag{15}$$

It can be observed that if we use backward Euler method but use $\Delta t/2$ as the step size, the coefficient matrixes of (13) and (14) will be the same. Therefore multi-RHS solvers can be used to solve both (13) and (14) with a little increase of computing time. The difference between $\boldsymbol{X}^k$ and $\boldsymbol{X}_*^k$ can be used to control the step size to prevent the solution of Crank-Nicolson method unstable.

B. Including Coreloss in System Equations

In the coreloss analysis in frequency domain, which is mostly used for laminated iron cores, the total coreloss per unit volume P_{core} can be expressed as:

$$p_{core} = k_e (f B_m)^2, \tag{16}$$

where, k_e is the coreloss coefficients, B_m is the peak induction and f is the frequency.

When this approach is applied to the time domain, the coreloss can be expressed as [1]:

$$P_{core} = \frac{1}{2\pi^2} k_e \left(\frac{\partial B}{\partial t}\right)^2. \tag{17}$$

Since, the coreloss per unit volume can also be expressed as:

$$P_{core} = H_e \cdot \frac{\partial B}{\partial t}. \tag{18}$$

Then, the equivalent magnetic field component H_e due to the coreloss can be expressed as:

$$H_e = \frac{1}{2\pi^2} k_e \frac{\partial B}{\partial t}. \tag{19}$$

The basic field equation can be expressed as:

$$\nabla \times (\nu \nabla \times \boldsymbol{A}) + \sigma \frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{J} + \nabla \times \boldsymbol{H}_c - \nabla \times \boldsymbol{H}_e, \tag{20}$$

where, $\boldsymbol{A}$ is the magnetic vector potential, $\boldsymbol{J}$ is the current density source, and $\boldsymbol{H}_c$ is the coercivity due to permanent magnets.

With (19), we can obtain the system equations with the coreloss included, as:

$$\begin{aligned}\nabla \times (\nu \nabla \times \boldsymbol{A}) + \sigma \frac{\partial \boldsymbol{A}}{\partial t} &= \boldsymbol{J} + \nabla \times \boldsymbol{H}_c - \nabla \times \left(k_e \frac{\partial \boldsymbol{B}}{\partial t}\right) \\ &= \boldsymbol{J} + \nabla \times \boldsymbol{H}_c - \nabla \times \left(k_e \frac{\partial \nabla \times \boldsymbol{A}}{\partial t}\right).\end{aligned} \tag{21}$$

III. EXAMPLES

A laminated transformer is used to verify the effectiveness of the proposed method. The voltage source $V_1 = 100\sin(2\pi \times 50t)$ is applied on the primary coil and the conductor number $N_p = 100$, $N_s = 50$, the resistances $R_p = 0.5\Omega$, $R_s = 0.2\Omega$, $R_l = 25.0\Omega$. The computed flux line distribution is shown in Fig. 1. The comparison results are shown in Table I. It is verified that the input power increase caused by coreloss effect can be precisely computed. Because no power unbalance in the time integration process is caused and the coreloss is included in the system equations, the final power balance error in the system is reduced significantly.

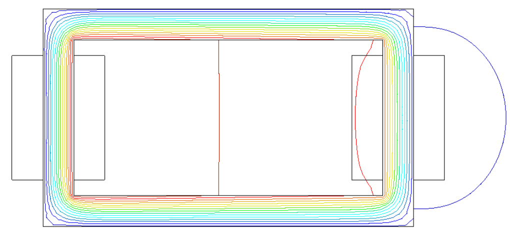

Fig.1. Flux line of a laminated transformer.

TABLE I COMPARISON OF DIFFERENT METHODS

Item	Core loss isn't included		Core loss is included	
Time step size	0.1 ms	0.01 ms	0.1 ms	0.01 ms
Input power (W)	49.6711	48.9727	52.9597	52.2628
Copper loss (W)	1.18912	1.18275	1.22266	1.21585
Coreloss (W)	0.0	0.0	3.28826	3.28976
Output power (W)	44.29264	44.32704	47.5472	47.5841
Power balance error (W)	4.189385	3.46296	0.901619	0.17315

IV. REFERENCES

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